

# Polarization, Political Instability and Active Learning\*

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## Abstract

In a multiperiod setting, decision-makers can learn about the consequences of their decisions through experimentation. We examine how polarization and political instability affect learning through experimentation. We distinguish two cases: (i) the decision to be made is not salient and does not affect the outcome of subsequent elections (exogenous elections) and (ii) the decision is salient and the election outcome depends on it (endogenous elections). It is shown that while the possibility of learning increases activism, the existence of political instability distorts learning. Furthermore, we demonstrate that, when elections are exogenous, polarization between political parties does not always decrease active learning.

*Keywords:* Active learning; elections; polarization

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## I. Introduction

The consequences of many decisions are surrounded by uncertainty. In a multiperiod setting, decision-makers can learn about the consequences of decisions through experimentation. Two types of learning through experimentation can be distinguished: passive and active. In the case of passive learning, the decision-maker's action is not induced by the possibility of learning. In the case of active learning, experimentation reduces the current utility of the decision-maker, but leads to information that can be used to improve future decision-making. There is a small but interesting literature on active learning. Early papers are Prescott (1972) and Rothschild (1974).

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The concept of active learning can be applied to several situations. For example, it can improve own understanding of how consumers make decisions about buying goods with uncertain quality, as in Grossman, Kihlstrom and Mirman (1977), or how firms ascertain the demand curves of their products, as in Rothschild (1974). Mirman, Samuelson and Schlee (1994) analyze active learning about the demand curve in duopolies, where a strategic element enters the analysis. In the context of economic policy, economists have studied the normative implications of active learning. Bertocchi and Spagat (1997) argue that the possibility of learning through experimentation provides a rationale for shock therapy in transition economies. In another paper, Bertocchi and Spagat (1993) study the implications of active learning for the optimality of money supply rules. So far, little attention has been paid to the implications of active learning for positive approaches to policy making.<sup>1</sup>

In this paper, we apply the concept of learning to decision-making in a political setting. More specifically, we address the question of how, in a two-party system, polarization and political instability affect learning through experimentation. To answer this question, we construct a highly stylized, two-period model of the behavior of a policy maker who has to make a decision about a public project. This project can be either implemented or rejected. Initially, the consequences of the project are uncertain. However, if the policy maker implements the project, she learns its consequences. In the next section, we show that when the policy maker is certain of remaining in office, the possibility of learning increases activism. This result is completely in line with the existing literature on active learning.

In Section III, we introduce elections into the model. The policy maker in period 1 faces an exogenous probability that in period 2 she is replaced by a policy maker with different preferences. We expected that uncertainty about the future preferences of the policy maker would decrease activism, as in Bertocchi and Spagat (1997). In general, however, this did not appear to be the case. When a policy maker who is biased against implementation is succeeded by a policy maker who is biased towards implementation, learning considerations may increase activism. The reason is that by implementing the project, the policy maker in period 1 can persuade her successor that the project should not be implemented in period 2. Thus experimentation is sometimes induced by a “let the other learn” effect. In Section IV, we endogenize elections. We show that if both policy makers attach equal value to being in office, at least one candidate chooses implementation of the project if the median voter prefers implementation to rejection. Consequently, in the case of the endogenous elections, learning is undertaken in the amount preferred by the median voter.

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<sup>1</sup>An exception is Bertocchi (1993), who uses results from the theory of active learning to provide an explanation for existing systems of public debt management.

The motivation for this paper is threefold. First, we want to gain an understanding of the impact of electoral competition on information gathering about policy consequences. This is important in the sense that the quality of policy often depends on the information underlying a policy decision. Second, in several countries, reforms are regarded as aimed at facilitating learning. The Netherlands, for example, intends to adopt a “performance” accounting system, in the hope that continuous evaluation of public policies will lead to better policy decisions. Studies, such as ours, may provide insight into the possible obstacles to desirable reforms of this type. Third, this paper contributes to the literature on political instability. Tabellini and Alesina (1990) show that political instability may give political parties incentives to create public debt. Peletier, Dur and Swank (1999) extend their analysis to public investment. Schultz (1996) argues that political polarization weakens information revelation when voters, but not parties, are uncertain about the working of the economy. Our paper shows that political instability may enhance active learning.

## II. Basic Model

We begin by introducing a simple two-period model, in which a policy maker has to make a decision about a public project. In each period  $t$ , the policy maker can choose between two alternatives: implementation, denoted by  $x_t = 1$ , and rejection, denoted by  $x_t = 0$ . When the policy maker chooses  $x_t = 1$ , her payoff is

$$U_t(x_t = 1|\mu) = p + \mu \quad (1)$$

where  $p$  denotes the policy maker’s predisposition towards the project, and  $\mu$  is a stochastic term, reflecting that the consequences of the project are surrounded by uncertainty. We assume that  $\mu$  is uniformly distributed on  $[-h, h]$ . When the policy maker rejects the project, her payoff is  $U_t(x_t = 0) = 0$  by normalization. Under full information, the policy maker would choose  $x_t = 1$  if  $\mu > -p$ . However, at the beginning of period 1, the policy maker does not observe  $\mu$ . Throughout this paper it is assumed that  $|p| < h$ . This assumption ensures that the realization of  $\mu$  determines whether or not the policy maker benefits from undertaking the project. As a consequence, the policy maker benefits from information about  $\mu$ . For notational simplicity, we assume that the policy maker does not discount the future. Her total utility is thus given by  $\sum_{t=1}^2 U_t$ . The policy maker can learn the value of  $\mu$  by implementing the project in the first period. If learning takes place, the decision about the project in period 2 is made under certainty.

Formally, the stages of the game can be described as follows:

- (i) Nature draws  $\mu$  from a uniform distribution with range  $[-h, h]$ .
- (ii) The policy maker makes a decision about the project in period 1:  $x_1 = 1$  or  $x_1 = 0$ .
- (iii) If  $x_1 = 1$ , nature reveals the value of  $\mu$  to the policy maker.
- (iv) The policy maker makes a decision about the project in period 2:  $x_2 = 1$  or  $x_2 = 0$ .

### *Solution of the Basic Model*

We now examine how the opportunity to learn affects the policy decision in period 1. More specifically, we derive the value of  $p$  for which the policy maker is indifferent between  $x_1 = 0$  and  $x_1 = 1$ . To ensure a time-consistent solution, we start by analyzing the second period.

In period 2, the decision about the project depends on the decision the policy maker has made in period 1. When  $x_1 = 0$ , the policy maker has not obtained information about  $\mu$ , and chooses  $x_2 = 1$  if and only if  $p > 0$ .<sup>2</sup> When  $x_1 = 1$ , the policy maker knows  $\mu$  and chooses  $x_2 = 1$  if and only if  $\mu > -p$ .

Now consider the policy maker's decision about the project in period 1. Anticipating her decision about the project in period 2, the expected payoff to the policy maker when she chooses  $x_1 = 1$  is

$$\begin{aligned} p + \Pr(\mu > -p)[p + E(\mu|\mu > -p)] &= p + \frac{1}{2h}(h+p)\left[p + \frac{1}{2}(h-p)\right] \\ &= \frac{1}{4h}(p^2 + 6hp + h^2). \end{aligned} \quad (2)$$

When the policy maker chooses  $x_1 = 0$ , her expected payoff is 0, if  $p \leq 0$ , and her expected payoff is  $p$ , if  $p > 0$ . Since the second term on the RHS of the first row in (2) is positive (expected utility in period 2 is greater than zero),  $x_1 = 1$  yields a higher expected payoff than  $x_1 = 0$ , if  $p > 0$ . Hence, if  $p > 0$ , then  $x_1 = 1$ . When  $p \leq 0$ ,  $x_1 = 1$  yields a higher expected payoff than  $x_1 = 0$  if the last expression in (2) is positive, implying:<sup>3</sup>

$$p > p^I = (-3 + 2\sqrt{2})h, \quad (3)$$

<sup>2</sup>Without loss of generality, we assume that when the policy maker is indifferent between  $x_i = 0$  and  $x_i = 1$ , she chooses  $x_i = 0$ .

<sup>3</sup>The last expression in (2) is also positive if  $p < (-3 - 2\sqrt{2})h$ . However, our assumptions concerning the values of  $p$  and  $h$  exclude this solution.

where  $p^f$  denotes the predisposition of a policy maker who is indifferent between  $x_1 = 0$  and  $x_1 = 1$ . There are two alternative ways of interpreting  $p^f$ . First, we can interpret  $p^f$  as indicating the type of policy makers, in terms of their predisposition towards a *given project*, who choose  $x_1 = 1$ . Second, we can interpret  $p^f$  as indicating the type of projects, in terms of their attractiveness, which are implemented by a *given policy maker*. According to the first interpretation, a decrease in  $p^f$  implies that more policy makers choose  $x_1 = 1$ . The second interpretation implies that when  $p^f$  decreases, more projects are implemented by the policy maker.

It is easy to see from (3) that  $p^f < 0$ . The implication is that a policy maker who is biased against implementation may choose  $x_1 = 1$ . On the basis of (3), we can make a clear distinction between two well-known concepts in the literature on learning. Passive learning takes place if  $p > 0$ : the opportunity to learn affects the policy decision in period 2, but not the policy decision in period 1. Active learning takes place if the opportunity to learn affects the policy in period 1. This occurs if  $p \in [(-3 + 2\sqrt{2})h, 0]$ . When there is no scope for learning (for instance, because a project cannot be repealed),  $x_1 = 0$  if  $p < 0$ .

Equation (3) implies that  $p^f$  decreases with  $h$ . Thus, the higher the level of uncertainty about the consequences of the project, the less restrictive is (3). The intuition behind this result is that the benefits of learning increase with uncertainty. When policy makers are risk averse, the benefits of active learning are even higher. However, with risk aversion, more uncertainty directly reduces the attractiveness of implementation.

### III. Uncertainty about the Preferences of the Future Policy Maker

Let us now introduce elections into the basic model in order to analyze the effect of elections on the condition for active learning. In the augmented model, elections are held at the end of period 1. Two policy makers compete for office: policy maker  $P$ , whose preferences are described by (1), and policy maker  $R$ , whose preferences are described by  $U_t^r(x_t = 1|\mu) = r + \mu$  and  $U_t^r(x_t = 0) = 0$ . The parameter  $r$  ( $|r| < h$ ) denotes policy maker  $R$ 's predisposition towards the project. The absolute value of the deviation of  $p$  from  $r$  can be interpreted as a measure of polarization. In period 1, policy maker  $P$  is in office. The probability that  $P$  stays in office in period 2 is denoted by  $\pi$ . The probability that  $R$  wins the elections is therefore given by  $1 - \pi$ . In this section, we make the strong assumption that  $\pi$  is exogenous. The motivation for this assumption is that policy makers make numerous decisions that do not receive attention in the media. It is unlikely that such decisions affect voter behavior. In the case of salient policy decisions, the

assumption that  $\pi$  is exogenous seems implausible. For this reason, we relax this assumption in Section IV.

### *Solution of the Augmented Model*

As before, we examine the scope for active learning by identifying the policy maker who is indifferent between  $x_1 = 0$  and  $x_1 = 1$ . Again  $p^I$  characterizes this policy maker.

**Proposition 1.** *If  $p < r$  and  $r \neq 0$ , then  $p^I$  is increasing in  $r$ . If  $p > r$  and  $r \neq 0$ , then  $p^I$  is decreasing in  $r$ .*

The proofs of this and the other propositions are in the Appendix.

Proposition 1 implies that  $p^I$  increases with  $|p - r|$ . Hence, when  $r \neq 0$ , a higher degree of polarization reduces the incentive to learn actively. The intuition behind this result is straightforward. Learning is attractive for  $P$  because it enables her to make an informed decision in the future. A higher degree of polarization increases the probability that a decision made in period 2 will not accord with  $P$ 's preferences. Thus, uncertainty about the preferences of the future policy maker generally reduces the benefits of learning and weakens the incentive to learn. In our model, learning requires implementation of projects. Hence, more polarization implies that implementation of projects becomes less attractive.

There remains the analysis of the effect of an increase in  $r$  on  $p^I$  when  $r = 0$ , i.e., in the case where  $R$  has no *ex ante* bias towards or against the project.

**Proposition 2.** *The effect of  $r$  on  $p^I$  is not smooth at  $r = 0$ :*

$$\lim_{r \uparrow 0} p^I(r) = p^I(r = 0) > \lim_{r \downarrow 0} p^I(r).$$

Proposition 2 indicates that the incentive for active learning is stronger when policy maker  $R$  is marginally biased towards implementation of the project than when he is marginally biased towards rejection of the project. To understand Proposition 2, suppose that policy maker  $R$  with  $r > 0$  is in office in period 2. If  $x_1 = 0$ ,  $R$  always prefers  $x_2 = 1$  to  $x_2 = 0$ . If  $x_1 = 1$ , then  $R$  only chooses  $x_2 = 1$  if  $\mu > -r$ . Thus, by implementing the project in period 1, policy maker  $P$  reduces the probability that policy maker  $R$  will choose  $x_2 = 1$  if  $R$  wins the election. This increases the utility of  $P$ , because  $p < 0$ . In addition to this “let the other learn” effect, policy maker  $P$  learns  $\mu$  herself by choosing  $x_1 = 1$ . Note that the smaller is  $r$  ( $r > 0$ ), the higher is the probability that  $x_1 = 1$  will induce  $R$  to choose  $x_2 = 0$ . As a consequence, the “let the other learn” effect is decreasing in  $r$  (given  $r > 0$ ).

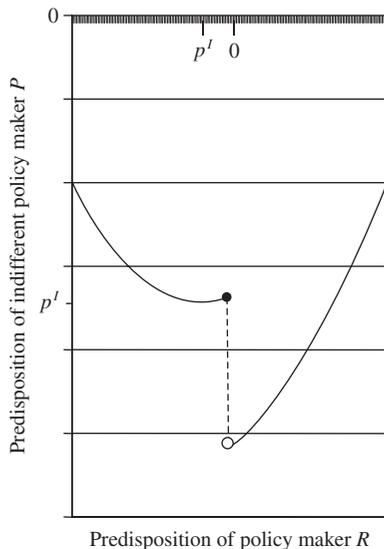


Fig. 1. Effect of polarization on active learning

Figure 1 summarizes our result about the effect of polarization on active learning. In general, a marginal increase in the degree of polarization reduces policy maker  $P$ 's willingness to choose  $x_1 = 1$ . However,  $p^I$  exhibits a negative jump at  $r = 0$ . The importance of this jump for learning is illustrated by the next proposition.

**Proposition 3.**  $\lim_{r \downarrow 0} p^I(r) < (-3 + 2\sqrt{2})h$ .

Proposition 3 indicates that the “let the other learn” effect is quite strong. In the case where  $R$  has a marginal bias towards implementation of the project, the incumbent is more willing to learn actively than in the benchmark case. Thus, there exists an interval of  $r > 0$ , in which political instability enhances active learning.<sup>4</sup>

In sum, for non-salient policy issues, the influence of electoral competition on active learning depends on the relative biases of the policy makers who compete for office. When both policy makers are biased towards rejection of the project, political instability always reduces the incentive to learn. When the incumbent’s opponent has a bias towards implementation, political instability may increase the incentive of the incumbent to learn actively. In both cases, however, a higher degree of polarization results in less learning. In

<sup>4</sup>In Bertocchi and Spagat (1997) political instability always dampens the learning effect. The reason is that in their model, the policy decision is not binary but defined on a continuum.

the next section we analyze how political instability and polarization affect learning through experimentation when policy revolves around a salient policy issue.

#### IV. Endogenous Elections

We now relax the assumption that the election outcome is exogenous. Moreover, we allow voters to choose the policy maker in period 1. Thus, two elections are held; the first determines the policy maker in period 1 and the second determines the policy maker in period 2. At the elections, each voter casts her ballot for the candidate whose policy yields the highest expected utility. Voter  $v$ 's preferences are described by  $u_t^v(x_t = 1|\mu) = v + \mu$  and  $u_t^v(x_t = 0) = 0$ . We assume a continuum of voters in terms of  $v$ . It is easy to see that in this setting the median voter's vote is decisive. Throughout, we assume that  $\pi = \frac{1}{2}$  if the median voter is indifferent between  $P$  and  $R$ . Let  $m$  denote the median voter's predisposition towards the project. We assume that  $m$  is common knowledge and that  $m > (-3 + 2\sqrt{2})h^5$  and  $p < m < r < 0$ . The implication of the first condition is that the median voter prefers active learning. The second condition states that, like both policy makers, the median voter is negatively predisposed towards the project. The reason is that we focus on active learning.

As shown below, with endogenous elections, the election outcome in period 2 depends on policy in period 1. As a consequence, the policy maker in period 1 can influence her probability of re-election. We assume that policy makers receive (ego) rents from holding office. More specifically, we add  $\lambda^i d_t^i$  to the utility function of each policy maker, where  $d_t^i$  takes the value one if policy maker  $i$  holds office in period  $t$  and takes the value zero otherwise. The parameter  $\lambda^i$  is a measure of how much value policy maker  $i$  attaches to holding office.

##### *Solution of the Model with Endogenous Elections*

Let us first determine policy in period 2. Since  $p < r < 0$ ,  $x_1 = 0$  implies  $x_2 = 0$ . It follows directly that the median voter is indifferent between  $P$  and  $R$  when  $x_1 = 0$ . When  $x_1 = 1$ ,  $P$  ( $R$ ) chooses  $x_2 = 1$  if and only if  $\mu > -p$  ( $\mu > -r$ ). This is anticipated by the median voter at the second election. As the median voter prefers  $x_2 = 1$  to  $x_2 = 0$  if and only if  $\mu > -m$ , the following result holds.

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<sup>5</sup>The term on the RHS of the expression is equal to the predisposition of the policy maker, who is indifferent between undertaking learning and not doing so in the basic case without the elections.

**Lemma 1.** Suppose  $x_1 = 1$ . Then (1) if  $\mu \in (-r, -m]$ ,  $P$  wins the elections; (2) if  $\mu \in (-m, -p]$ ,  $R$  wins the elections; (3) for all other values of  $\mu$ ,  $P$  wins the elections with the probability  $\frac{1}{2}$ .

Figure 2 illustrates Lemma 1 graphically.

Let us now consider policy in period 1. Suppose  $R$  is in office. When  $R$  chooses  $x_1 = 0$ , his expected payoff equals  $1\frac{1}{2}\lambda^R$ . When he chooses  $x_1 = 1$ , his expected payoff equals:

$$\begin{aligned} & r + \lambda^R + \Pr(\mu \in (-m, h])(r + E(\mu | \mu \in (-m, h])) \\ & + \lambda^R [\Pr(\mu \in (-m, -p]) + \frac{1}{2}\Pr(\mu \in [-h, -r]) + \frac{1}{2}\Pr(\mu \in (-p, h])] \\ & = r + \lambda^R + \frac{1}{2h}(h+m)(r + \frac{1}{2}(h-m)) + \frac{1}{2h}\lambda^R[h+m - \frac{1}{2}(r+p)]. \end{aligned} \quad (4)$$

When  $\lambda^R = 0$ ,  $R$  prefers  $x_1 = 1$  to  $x_1 = 0$  since  $r > m > (-3 + 2\sqrt{2})h$ . Policy maker  $R$  prefers  $x_1 = 0$  to  $x_1 = 1$  if:

$$m < \frac{1}{2}(r+p) \quad (5)$$

and

$$\lambda^R \geq \frac{2hr + (h+m)[r + \frac{1}{2}(h-m)]}{\frac{1}{2}(r+p) - m} \quad (6)$$

Condition (5) shows that  $R$ 's choice of  $x_1 = 0$  requires the preferences of the median voter to be closer to the preferences of  $P$  than to those of  $R$ . The implication of (5) is that  $x_1 = 1$  is unattractive for  $R$  from an electoral point

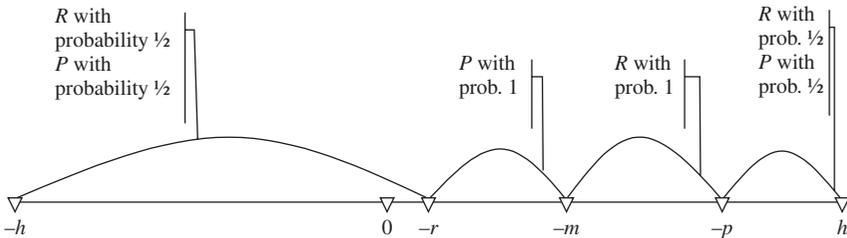


Fig. 2. Winners of the elections depending on the value of  $\mu$

of view. If, in addition to (5),  $R$  cares sufficiently about holding office, then he chooses  $x_1 = 0$ .

Now suppose  $P$  is in office. Along the same lines we can show that  $P$  chooses  $x_1 = 0$  if

$$p + \lambda^P + \frac{1}{2h}(h+m)(p + \frac{1}{2}(h-m)) + \frac{1}{2h}\lambda^P[h-m + \frac{1}{2}(r+p)] \leq 1 \frac{1}{2}\lambda^P \quad (7)$$

It is easy to see that when condition (5), which is a necessary condition for  $R$  to choose  $x_1 = 0$ , holds,  $P$  chooses  $x_1 = 0$  if

$$\lambda^P \leq \frac{-2hp - (h+m)[p + \frac{1}{2}(h-m)]}{\frac{1}{2}(r+p) - m}. \quad (8)$$

Thus, when the preferences of the median voter are closer to the preferences of  $P$  than to those of  $R$ ,  $P$  chooses  $x_1 = 0$  if she does not care sufficiently about holding office. Combining (5), (6) and (8) yields the following proposition.

**Proposition 4.** *Suppose  $m > (-3 + 2\sqrt{2})h$  and  $\lambda^P = \lambda^R = \lambda$ . Then at least one policy maker chooses  $x_1 = 1$ .*

Proposition 4 states that when a majority of voters prefer active policy, at least one policy maker chooses active policy. The intuition is as follows. When the preferences of the median voter are closer to the preferences of  $R$  than to those of  $P$ , it is always optimal for policy maker  $R$  to choose  $x_1 = 1$  if in office in period 1. By implementing the project,  $R$  maximizes his partisan utility and, at the same time, increases the probability of being elected in period 2 (see Figure 2). However, when the preferences of the median voter are closer to the preferences of  $P$  than to those of  $R$ , implementation of the project increases the probability that the median voter will vote for  $P$  in period 2. Then, if the rents from being in office are large, electoral considerations force  $R$  to choose  $x_1 = 0$  if in office. However, the same electoral considerations induce  $P$  to choose  $x_1 = 1$  if in office. Hence, when policy revolves around a salient issue, polarization is not an obstacle to learning. Of course, if  $\lambda^R$  is high and  $\lambda^P$  is low, then both policy makers might choose  $x_1 = 0$ , while the median voter prefers  $x_1 = 1$ .

## V. Concluding Remarks

Learning through experimentation—or active learning—occurs when an agent makes a decision that reduces current utility, but leads to information

that is expected to improve future decision-making. In this paper, we have addressed the question of how polarization and political instability affect learning through experimentation in a two-party system where a policy maker with different preferences may succeed the incumbent. Two cases were distinguished.

First, we have analyzed a model in which a policy maker has to make a binary decision about a project, which does not affect her chances of re-election. We have shown that in this case, a higher degree of polarization generally reduces the incumbent's incentive to learn through experimentation. However, the relationship between polarization and learning through experimentation is not continuous. It matters whether the successor is biased against or towards implementation. When the successor is biased towards implementation, the incumbent may have a stronger incentive to implement the project than when the successor is biased against the project.

Next, we have analyzed a model in which elections revolve around the project under consideration. We have shown that when the two candidates value holding office to the same extent, then at least one of the two candidates will choose activism if a majority of voters prefers activism.

Our analysis is based on several restrictive assumptions, some of which are made for simplification and are innocuous. For example, we have assumed that policy makers do not discount the future and that once a project has been implemented, its consequences are known. Relaxing these assumptions does not affect our results qualitatively. Two other assumptions are less innocuous. First, our model revolves around a single project. As a consequence, voters evaluate candidates on the basis of a single issue. In reality, policy makers make decisions about numerous projects, both old and new. It is easy to show that when policy makers have to make decisions about more than one project, voters sometimes face a trade-off between learning about new projects and repealing unfavorable, old projects. Second, our model focuses on projects that can either be implemented or not. Policy makers often have to make binary decisions. However, it is unclear whether our results generalize to decisions about continuous variables.

Though our results are derived from a highly stylized model, we believe that they are important for two reasons. First, our analysis provides insight into the way polarization and political instability affect policy making under uncertainty. Second, our results have normative implications. Nowadays, researchers are considering ways to transform the public sector into a learning organization (OECD, 1999). Our analysis points out that polarization and political instability are potential obstacles to such a transformation.

**Appendix**

*Proof of Proposition 1*

It is easy to verify that  $P$  always prefers  $x_1 = 1$  to  $x_1 = 0$  if  $p > 0$ . Moreover if  $x_1 = 1$ , then in period 2  $P$ , if elected, chooses  $x_2 = 1$  iff  $\mu > -p$ , and  $R$ , if elected, chooses  $x_2 = 1$  iff  $\mu > -r$ . Note also that if  $x_1 = 0$ , then  $x_2 = 1$  occurs iff  $R$  is elected and  $r > 0$ .

Suppose  $p < 0$  and  $r < 0$ . Then,  $x_1 = 1$  yields a higher total payoff to  $P$  than  $x_1 = 0$  if

$$p + \pi \Pr(\mu > -p)[p + E(\mu|\mu > -p)] + (1 - \pi) \Pr(\mu > -r)[p + E(\mu|\mu > -r)] > 0.$$

Thus,  $P$  is indifferent between  $x_1 = 1$  and  $x_1 = 0$  if

$$\frac{1}{4h}(\pi p^2 + [6h + 2(1 - \pi)r]p + h^2 - (1 - \pi)r^2) = 0. \tag{A1}$$

Equation (A1) has a unique solution for  $p, p^1_{r < 0}$ , in the interval  $[-h, 0)$ . Application of the implicit function theorem shows that  $\partial p^1_{r < 0} / \partial r > 0$  if  $p < r$  and  $\partial p^1_{r < 0} / \partial r < 0$  if  $p > r$ .

Now suppose  $p < 0$  and  $r > 0$ . Then  $x_1 = 1$  yields a higher payoff to  $P$  than  $x_1 = 0$  if the LHS of (A1) is larger than  $(1 - \pi)p$ . This implies that  $P$  is indifferent between  $x_1 = 1$  and  $x_1 = 0$  if:

$$\frac{1}{4h}(\pi p^2 + [2h(1 + 2\pi) + 2(1 - \pi)r]p + h^2 - (1 - \pi)r^2) = 0. \tag{A2}$$

Like (A1), (A2) has a unique solution for  $p, p^I_{r > 0}$ , in the interval  $[-h, 0)$ . Application of the implicit function theorem shows that  $\partial p^I_{r > 0} / \partial r > 0$ . Q.E.D.

*Proof of Proposition 2*

When  $r$  goes to zero, the LHS of (A1) approaches  $(\pi p^2 + 6hp + h^2)/4h$  and the LHS of (A2) approaches  $(\pi p^2 + 2(1 + 2\pi)hp + h^2)/4h$ , so that

$$\lim_{r \downarrow 0} p^I_{r < 0} = p^I(r = 0) = \frac{(-3 + \sqrt{9 - \pi})h}{\pi} \tag{A3}$$

and

$$\lim_{r \downarrow 0} p^I_{r > 0} = \frac{(-1 - 2\pi + \sqrt{1 + 3\pi + 4\pi^2})h}{\pi}. \tag{A4}$$

It is easy to show that, for  $0 < \pi < 1$ , the expression in (A3) is larger than the expression in (A4). Q.E.D.

*Proof of Proposition 3*

It is straightforward to show that the inequality  $(-1 - 2\pi + \sqrt{1 + 3\pi + 4\pi^2})h/\pi < (-3 + 2\sqrt{2})h$  holds for all  $\pi \in (0, 1)$ , and becomes an equality at  $\pi = 1$ . Q.E.D.

*Proof of Proposition 4*

After substituting  $\lambda^P = \lambda^R = \lambda$  into (6) and (8) and combining the two, we obtain that both policy makers choose for  $x_1 = 0$  if (5) and

$$\frac{2hr + (h + m)[r + \frac{1}{2}(h - m)]}{\frac{1}{2}(r + p) - m} \leq \lambda \leq \frac{-2hp - (h + m)[p + \frac{1}{2}(h - m)]}{\frac{1}{2}(r + p) - m}. \quad (\text{A5})$$

Expression (A5) can be simplified to

$$r + p \leq \frac{-(h^2 - m^2)}{3h + m}. \quad (\text{A6})$$

It is easy to show that in order for (A6) and (A5) to hold,  $(-3 - 2\sqrt{2})h \leq m \leq (-3 + 2\sqrt{2})h$  must be true. This condition, however, contradicts our assumption that  $m > (-3 + 2\sqrt{2})h$ . Q.E.D.

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